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RETAINING WALLS IN ANCIENT THEATER CONSTRUCTION - GEOTECHNICAL APPROACH

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Abstract. High retaining walls (*αναλήμματα*) were necessary for the construction of ancient theatres. They were most probably constructed following technics that evolved for the construction of high retaining walls necessary for high terraces and fortifications. Tightly fitting dry stone plinths were used for the walls. It was apparently understood that backfilling the retaining walls with rock fragments instead of soil materials reduced earth pressures and allowed free drainage. A further improvement was achieved by making the hidden face of the wall rough with protruding building stones. This “roughness” improves wall stability. External buttresses were not preferred but were used in some cases to improve stability. Internal hidden buttresses linked to the wall were used to allow high wall construction. Finally, internal buttresses were connected to create compartments that allowed safe construction of very high walls. There is also, still undocumented by archaeological excavations, reference to reinforcement of the backfill material with horizontally placed timber beams vertical to the wall. These methods indicate good empirical knowledge of earth pressure processes. The article assumes some typical dimensions for retaining structures and examines the improvement provided by each of the above techniques. From five (5) to nineteen (19) meter high structures were examined. Elementary earth pressure theory is initially used, followed by 3-D finite element analysis to investigate improvements in safety factors and maximum safe heights. The analyses assumed as a reference a 5m high wall, backfilled with rockfill. The inclusion of long protrusions into the backfill increases the available safety factor by ~30%. By incorporating a typical pattern of external buttresses, the maximum safe height of the wall increases by ~2m. Using a typical pattern of internal buttresses allows construction of up to 14m high walls. Finally, using compartmentalization allows construction of up to 19m high walls. The maximum achievable wall heights of the performed analyses agree broadly with the heights observed in the retaining walls of ancient theatres.

Keywords: ancient theatres, earth pressures, retaining walls, FEA

1 Introduction

The main ancient source of information on the design and construction of ancient theaters is the monumental work by the Roman architect and military engineer Marcus Vitruvius Pollio «De Architectura», late 1st century BCE. No other ancient sources survive on the matter.

In Book V, Chapters VI and VII he describes the design of Roman and Greek theaters. In Book III, Chapter IV, he describes the foundations of temples and in Book VI, Chapter VIII, he refers to foundations and substructures. Of special interest is paragraph 7 of Chapter VIII, where he discusses retaining walls.

The ancient Greek theater seems to have been finalized its semi-circular arrangement of the *koilon* (*cavea*) and the circular orchestra sometime between the 5^{th} and the 4^{th} century BCE. Previous arrangements were mainly adjusted to available space having different shapes and minimizing earthworks. The timber seats were gradually replaced by stone seats. Important contribution to the research on the evolution of the ancient theater was provided in the conference “The Architecture of the Ancient Greek Theatre” (Frederiksen, Gebhard & Sokolicek 2015).

An important source of information on the ancient theaters is provided by the DIAZOMA publications (https://diazoma.gr), where the work of numerous archaeologists and engineers is presented.

The realization on the ground of the finalized semi-circular form demands larger and more complicated earthworks, i.e. excavations, retaining structures and backfilling. To complete the sides of the koilon, high retaining structures were needed. Different and often difficult geological conditions had to be faced. The engineering solutions portray a high-level empirical knowledge of basic soil mechanics principles, applied in high retaining wall construction.

Locally available materials are normally used for construction and “rich” theaters would bring better quality stone, usually limestone or marble, from large distances.

2 Construction of retaining structures

The construction of retaining walls is a major subject of Soil Mechanics and Geotechnical Engineering. Their construction followed empirical rules up to the establishment of the earth pressure theory by Coulomb towards the end of the 18th century and Rankine in mid-19th century. Seismic earth pressures were examined much later during the 1920’s. The importance of geological conditions and the role of the Geotechnical Engineer in the study and restoration of ancient monuments has been recognized early on (Kerisel 1975; Calabresi 2013).

The impressive retaining walls of the ancient theaters imply a good empirical knowledge of geotechnical engineering that ancient engineers possessed. Very large retaining walls were built before in Mesopotamia and Egypt, most impressive being the walls of Babylon. Bilis (2019) doctoral thesis deals with construction of retaining structures in antiquity.

High walls were necessary for the retaining of the fills required for the formation of the *koilon* at the sides and frequently at the back of the theater. Walls with a height more than 10m were frequent, occasionally approaching 20m (Dodona theater).

Although the rules that were followed are not known to us, observing the ancient structures indicates that they had a good knowledge of safe foundation, reduction of earth pressures and improving the retaining ability of the structure. Vitruvius in Book VI, Chapter VIII Foundations, describes retaining walls for substructures. The same principles were used for retaining walls above ground. Vitruvius describes the use of external buttresses and internal compartmentalization as means of improving stability. He emphasizes that the use of soil-like materials for backfill absorb water and increase earth pressures, often with destructive consequences. It is therefore implied, although not directly stated, that use of rockfill materials would be preferable as backfill.

Belenis (2012) presents the retaining walls of the early Hellenistic era at the theaters of Filippi and Aiges. He discusses evidence for the use of internal hidden counterweights, created by protruding stone beams into the backfill of the wall. These beams are created by elongated strong stones that are part of the wall construction. Belenis further postulates that retaining wall safety could have been improved by the inclusion of horizontal timber beams vertical to the wall. No archaeological evidence exists so far.

The retaining walls were made as a rule by strong stones formed and well fitted together without mortar. These walls could only sustain compressive stress.

Exception to the rule of stone construction presents the Hellenistic phase of the ancient Demetrias theater where mudbricks (not fired) were used. Furthermore, backfilling was made with the same material as the one used for mudbricks. This material is watertight and has considerable cohesion. When properly protected and maintained, mudbrick structures can last for a very long time.

Summarizing, the methods of improving stability and allowing progressively higher retaining structures are:

<u>Backfill materials</u>

Use of rockfill for backfilling reduces earth pressures and allows drainage behind the wall. This is observed in most Hellenistic theaters, where rock fragments were available.

<u>Rough internal face of the wall</u>

A method used practically everywhere. The roughness of the hidden face of the wall is achieved by not curving the inside face of the stones. It is not known whether this was done intentionally to increase roughness or to save manpower. A rough internal face means that the friction of the wall to backfill interface will be equal to the friction angle of the backfill. This adds stability, compared to smooth surfaces (like modern concrete retaining walls) where the interface shear strength is less than that of the backfill.

<u>Internal intrusions (counterweights)</u>

This method, reported by Belenis (2012), is schematically presented in Figure 1a. These intrusions link the backfill to the wall, increasing its operational width and considerably improving the stability of the wall. The method of making the backfill contributing to the stability of the wall is extensively used today by various methods.

<u>External buttresses</u>

Construction of external buttresses at a spacing and height varying according to the wall height improve the stability of the wall (Figure 1b). This method was known but not favored by ancient Greeks and was used sparingly. Occasionally they have an arched shape and butt against neighboring buildings as in the Filippi theater (Koukouli-Chrysanthakaki & Karadedos 2012). A form of monumental external buttressing is observed at the Dodona theater, where tower like structures filled with layered stone slabs support a wall with an estimated height of 20m.

<u>Internal buttresses</u>

Construction of internal hidden buttresses considerably improve the wall stability and allow construction of higher walls (Figure 1c). Internal buttresses have been observed in many ancient theaters, notably at the Dionysos theater in Athens (Samara 2012), and at Messene (Themelis 2010; Yoshitak 2021).

<u>Compartmentalization</u>

Frequently the internal buttresses were linked together forming compartments as described by Vitruvius (Book VI, Chapter VIII). This technique is observed in the ancient theaters of Dionysus in Athens (Samara 2012) and at Messene (Themelis 2010; Yoshitak 2021). It allows the construction of very high retaining walls (Figure 1d).

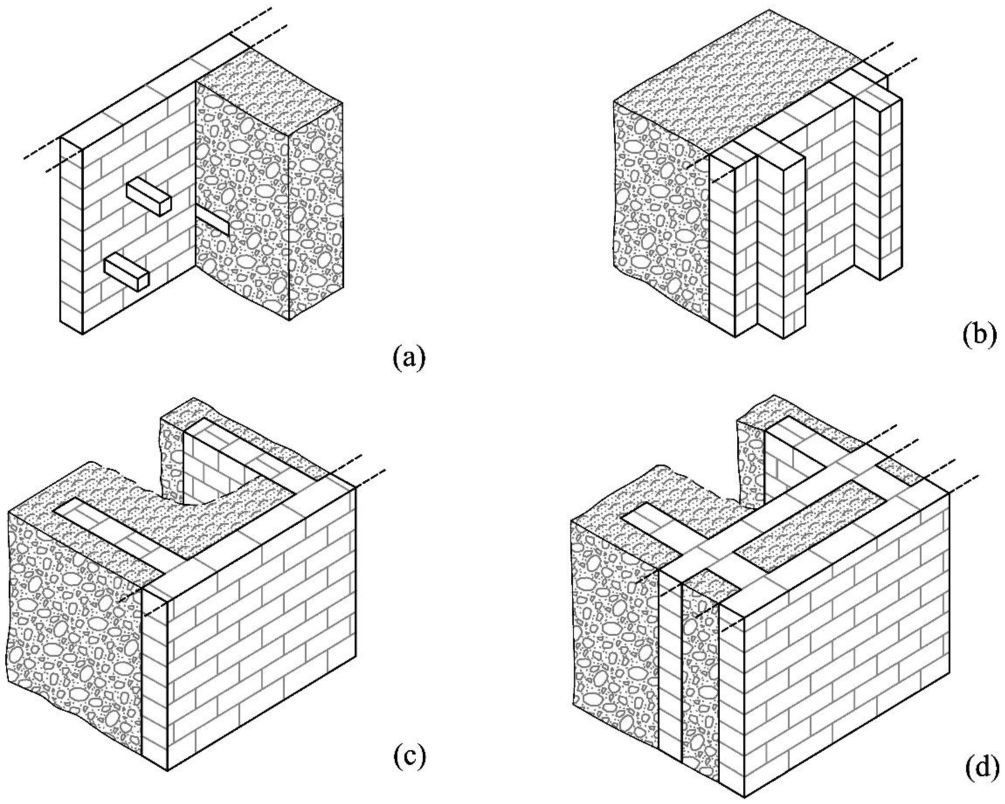

Figure 1. Methods used for increasing the retaining wall stability (a) Internal counterweights, (b) external buttresses, (c) Internal buttresses, and (d) compartmentalization.

3 Analyses of wall configurations

3.1 Effect of backfill material and wall roughness

Both the backfill material and the wall roughness are essential parameters that determine the maximum height that a wall of certain thickness can reach.

In the current study, in order to assess the effect of the backfill material, the influence of its shearing resistance is examined by applying the earth pressure theory (Rankine 1857; Terzaghi et al. 1996). Two (2) different backfill materials are examined, i.e., earth fill and rockfill. The effective cohesion (c') of both these materials is considered equal to 0kPa for the current study. The effective angle of shearing resistance (φ') is considered equal to 30^o for the earth fill and equal to 40^o for the rockfill. The unit weight (γ) is considered equal to 18kN/m^3 for the earth fill and 20kN/m^3 for the rockfill.

The developing vertical stresses ($\sigma'_{v,o}$) behind the wall are calculated as described below:

$$\sigma'_{v,o} = \gamma \cdot H$$

where, γ, the unit weight, and H, the height measured from the top of the ground downwards.

Then, based on the effective angle of shearing resistance (φ'), the coefficient of active pressure (K_a) is calculated for each backfill material, as described below:

$$K_{\alpha} = \tan^2\left(45 - \frac{\varphi'}{2}\right)$$

Finally, the developing horizontal stresses ($\sigma'_{h,a}$) behind the wall due to the backfill material are calculated for each backfill material, as described below:

$$\sigma'_{h,a} = K_a \cdot \sigma'_{v,o}$$

The resulted horizontal stresses by applying the above theory, are presented in Figure 2 for both the earth fill (solid line) and the rockfill (dashed line). For a given wall's height in the current example, i.e., 5m, the developed horizontal stresses due to rockfill are ~27.5% less than the ones due to earth fill. In order for the rockfill to develop the same level of horizontal stresses at the wall's base, the wall's height should be 1.9m higher, i.e., 6.9m.

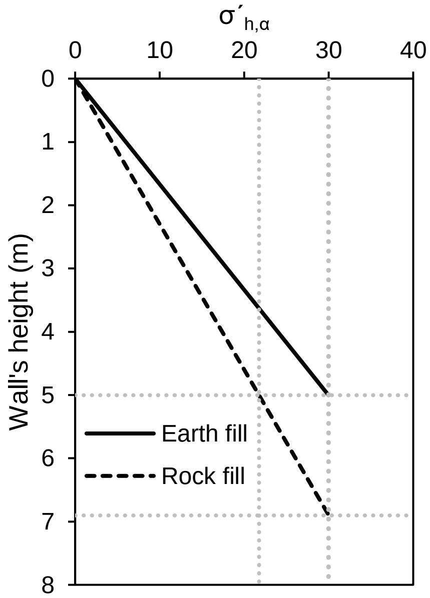

Figure 2. Developing horizontal stresses ($\sigma'_{h,a}$) behind the wall due to two (2) different materials (solid line: earth fill; dashed line: rockfill; grey dotted lines are shown for supervisory purposes).

The influence of the wall roughness is studied parametrically by employing the software BETONexpress ver. 09-09-2022 (RUNET software). This software is used for the dimensioning, among others, of gravity walls. Keeping the dimensions of the base (1.7m) and the crown (1.0m) of the wall fixed, the maximum height that can be achieve is examined by alternating the friction angle ($\varphi_{interface}$) of the interface between the wall and the filling material (rockfill for the parametric analysis herein). Three (3) different friction angles are studied, resulting on different maximum wall's height (H_{max}):

i. $\varphi_{interface} = 0^o$ | $H_{max} = 4.4m$,
ii. $\varphi_{interface} = 2/3 \cdot \varphi_{backfill}$, $\varphi_{backfill} = \varphi_{rock\ fill} = 40^o$ | $H_{max} = 5.7m$ (~29.5% increase from (a)), and
iii. $\varphi_{interface} = \varphi_{backfill} = \varphi_{rock\ fill} = 40^o$ | $H_{max} = 6.5m$ (~47.7% increase from (a)).

It is noted that case (i) is considered as an extreme scenario, while case (iii) represents a realistic scenario where the inside face of the wall is left rough.

3.2 Effect of different wall geometries

In order to assess the impact of different geometries (see Figure 1) on the maximum achievable wall's height (H_{max}), as well as on the available Safety Factor (SF), Finite Element Analyses (FEA) in three (3) dimensions are performed by employing the software Plaxis3D ver. 2.1 (Plaxis BV). Three (3) materials are used, same for all simulations, i.e., bedrock, wall and rockfill. Their mechanical properties are presented in Table 1. It is noted that the material model of the bedrock and the rockfill is elasto-plastic Mohr-Coulomb (M-C), while the wall is simulated as linear elastic (LE).

A fully rough interface is assumed between wall and backfill. The walls are assumed embedded in the foundation by 0.5m in all cases.

Table 1. Materials and their mechanical properties in Plaxis3D.

Material (model)	Bedrock (M-C)	Wall (LE)	Rockfill (M-C)
Unit weight, kN/m^3	22	25	20
Effective cohesion, kPa	30	-	1
Effective angle of shearing resistance, deg.	45^o	-	40^o
Young's modulus, MPa	500	500	50
Poisson's ratio	0.25	0.24	0.33

The different examined wall arrangements are attempting to follow some typically observed dimensions and they are not precisely copying existing walls. It should also be emphasized that the analysis is not accurately modelling the behavior of the wall, the foundation and the backfill. In that sense it is approximate and should be considered as such.

Initially, a simple wall was studied. The thickness of that wall was equal to 1m. Then, the results of this wall were used as reference values for comparing with the results of the other wall configurations. It is noted that in the current paper, the presented results are referred up to the maximum wall's height for which the numerical analysis could be completed. Beyond these heights, computation could not be completed. Details about the geometry of each model is presented below:

a) Simple wall | studied heights: 1, 2, 3, 4, 5m.
b) External buttresses | buttresses' dimensions: 1.0m × 1.0m (length × width), spacing: 6 m, studied heights: 5, 6, 7m.
c) Internal buttresses | buttresses' dimensions: 3.5m × 1.0m (length × width), spacing: 6 m, studied heights: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15m.
d) Compartmentalization | secondary walls' thickness: 1.0m, distance of external and internal walls: 4m, spacing of secondary transverse walls: 6m, studied heights: 5, 10, 14, 16, 18m.
e) Internal long protrusions | protrusions' dimensions: 1.0m × 0.3m × 0.3m (length × width × height), horizontal spacing: 2m, vertical spacing: 1m, studied heights: 5m.

Indicative images of the above models are presented in Figure 3.

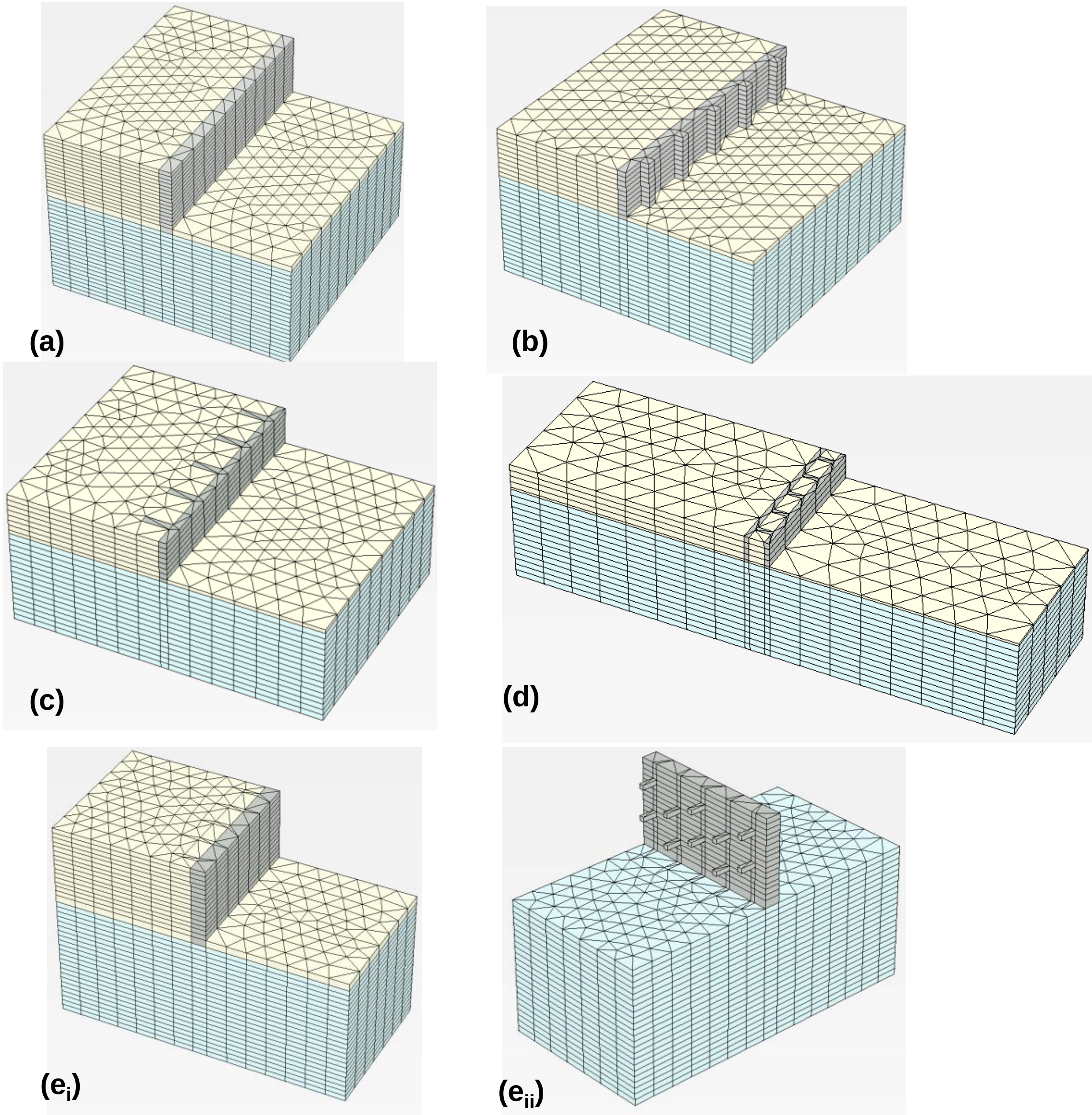

Figure 3. Indicative models of the construction methods of Figure 1 in Plaxis3D (walls' height: 5m) | (a) simple wall, (b) external buttresses, (c) internal buttresses, (d) compartmentalization, (e) internal long protrusions (i: whole model; ii: rear face of the wall without backfilling to show the internal protrusions).

4 Discussion on FEA results & Conclusions

In the current paper, five (5) different wall geometries are studied, as presented in Section 3.2. Figure 4 presents the FEA results of Section 3.2, illustrating the wall height (m) on the ordinate vs the achieved safety factor (SF) on the abscissa. A vertical dotted line is included as reference for the SF of the 5m high simple wall (SF_{sw5} = 1.461). Figure 5 illustrates the wall height (m) on the ordinate vs the percentage change of SF compared to SF_{sw5} on the abscissa. A vertical dotted line is included (0% change) as reference of SF_{sw5}.

In these Figures, the maximum achievable height of the wall is strongly influenced by the construction method. Compartmentalization achieves the best results both on the maximum achievable height (~19m) and the SF (~250% increase for the case of 5m high wall). On the other hand, the internal long protrusions resulted on the least SF increase (~31% increase for the case of 5m high wall). In essence, the internal protrusions act as a mean to increase the friction at the wall-backfill interface.

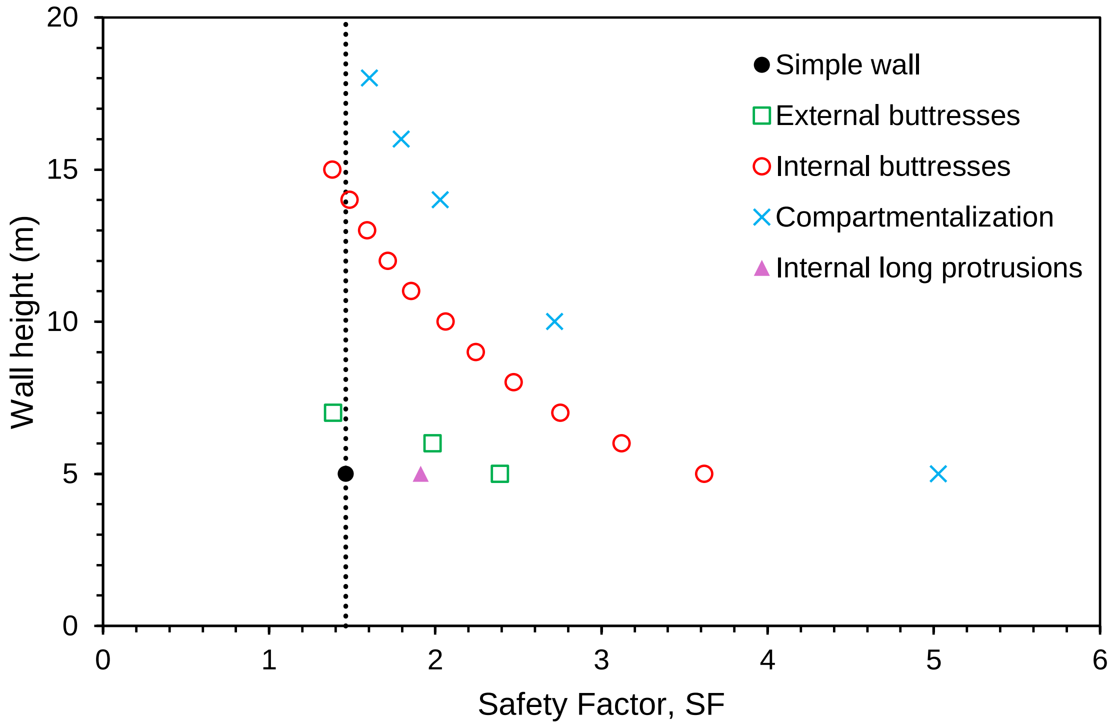

Figure 4. Wall height vs Safety Factor (SF) for the different construction methods. A vertical dotted line is included as reference for the SF of the 5m high simple wall (SF_{sw5} = 1.461).

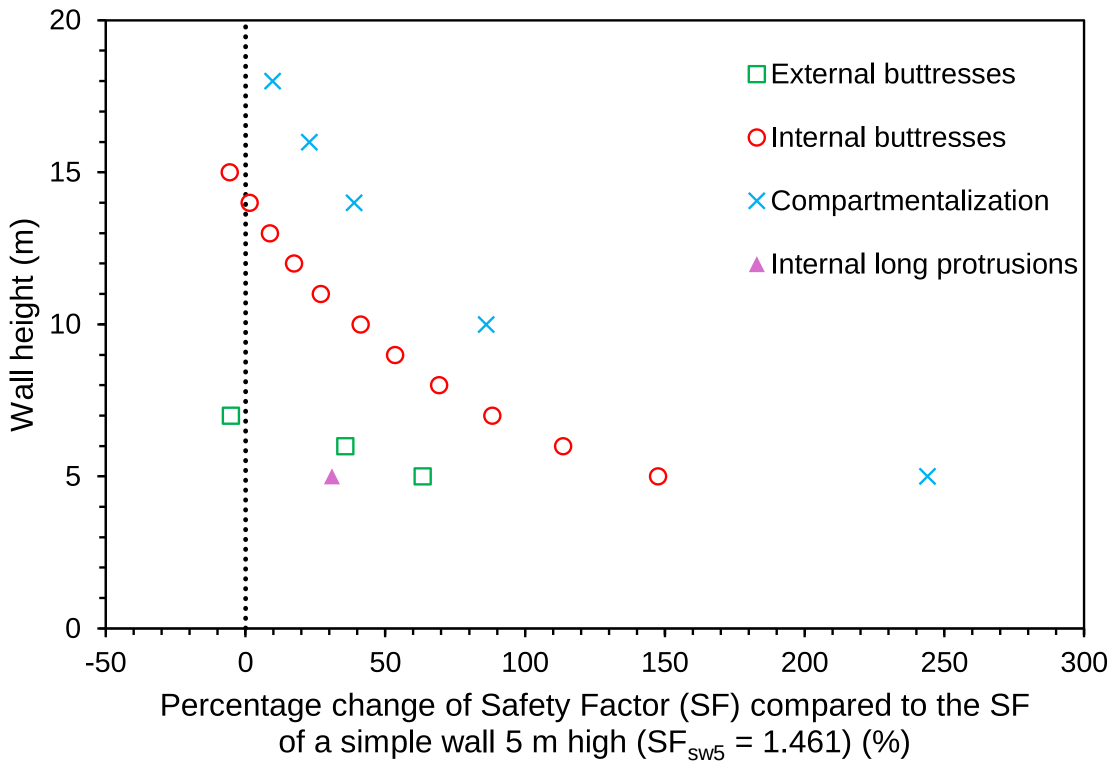

Figure 5. Wall height vs percentage change of Safety Factor (SF) for the different construction methods compared to the SF of a simple wall 5m high. A vertical dotted line is included as reference for the 5m high simple wall (0% change).

Comparing the simulation results of the buttresses, it becomes apparent that cases where internal buttresses were used, the retaining wall was able to achieve higher heights compared to the external ones.

The ancient retaining walls are made of tightly fitted stone plinths of various shapes. A precise very accurate method of modelling is to use a Discrete Element Method (DEM). The wall in the present analysis was modelled as continuous and linear elastic. The resulting stresses of the wall were examined to investigate probable tensile stresses that cannot be sustained by the actual wall and could put the results into doubt.

The stresses in the wall were always compressive. A example is illustrated in Figure 6 for the 5m simple wall, where the compressive stress reaches a maximum value of 470kPa at the base. This is reassuring that the approximate analyses are accurate, within reason. The stone plinths can easily undertake the maximum compressive stresses.

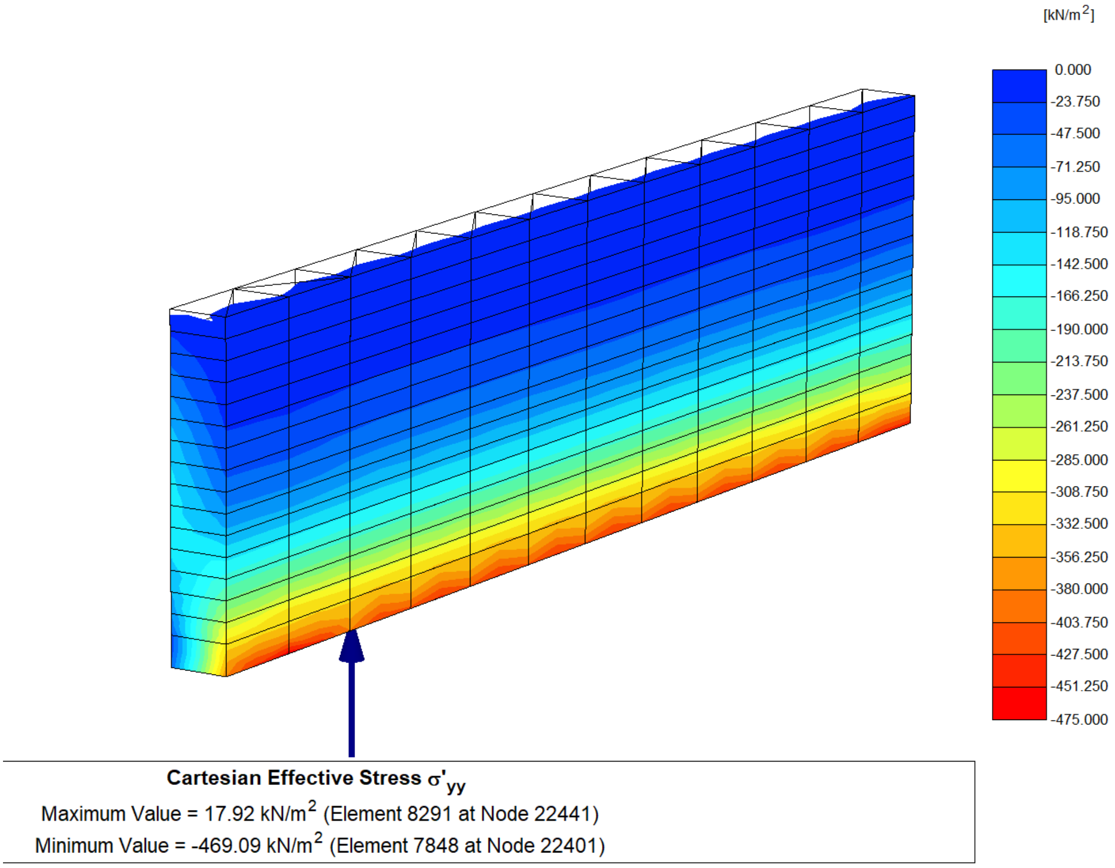

Figure 6. Vertical effective stress σ'_{yy} at the 5m high simple wall in Plaxis3D.

Finally, it is noted that the numerical simulations presented herein do not consider the seismic effect on the structures and it is suggested for further research.

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