ANALYSIS OF THE SUNDIALS ON THE "TOWER OF THE WINDS" IN ATHENS

(FIRST RESULTS BASED ON NEW DATA ATTAINED WITH A HIGH PRECISION GEODETIC DATA ACQUISITION SYSTEM)

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The "Tower of the Winds" in Athens, built by Andronikos Kyrrhestes, is one of the best preserved monuments from ancient times.

In the past the sundials on the Tower of the Winds have been analyzed by Delambre (1817), Drecker (1925), Antonacopoulos / Fragakis (1969) and Bromley/Wright (1989) based on the measurements done in 1762 by J. Stuart and N. Revett. Only Palaskas (1846) performed own measurements. The measurements in all these cases as is apparent now from the new measurements are not that accurate as they seem to be. Despite that all authors draw conclusions about the accuracy of the construction in a favourable way. Gibbs (1976) seems to be the first to having recognized the few remains on the cylindric annex as part of another sundial.

It may be noticed, that Vitruvius mentions the "Tower of the Winds" and its designer (book I,25) only in the context of finding the cardinal directions for town planning but not the Horologion. Again later (book IX) when he deals with sundials in particular no mention is done about this outstanding time standard. This aroused a discussion (Delambre) about whether the sundials were added later. Palaskas objected to this already, and if it is not already clear from the overall architectural concept, the design of the cylindrical sundial would be proof enough.

The current analysis of the sundials is part of a Greek-German Project (Kienast) to thoroughly investigate the whole building.The data acquisition was performed with the Leica TPS-System 1000 resulting in absolute coordinates with an uncertainty of less than 3 mm. The position and orientation of the coordinate system is also known to high precision. Taking into account the achievements in mathematics, astronomy and geography available at that time the analysis shall reveal the cardinal design parameters of the sundials as they are

- geographic latitude and orientation of the wall
- angle of ecliptic,
- length of the gnomon in ancient units.

It shall be pointed out here, that the "engineers" of that time faced a similar problem of metrology as must still be solved today in the national laboratories : when setting up a primary standard - the atomic clock nowadays - there is no simple standard available to which to compare. The duration of one day, when the shadow of the sundial arrived at the very same position (which is observable with an uncertainty of only a few seconds), could have been used for calibrating the water-clock.

Since the objective of the analysis is to find parameters used for the construction of given sundials, not to construct a sundial according to given parameters, one is faced with an "inverse problem" with the intrinsic difficulty that it may tend to be "ill conditioned" because of deficiencies in the available data. The "inverse problem" will not be solved in this case with a general approach but rather by knowledge guided trial and error. A set of parameters is selected in order to calculate a model which will be compared with the reality presented. The consequence of the deficiencies will be that two or more models are equivalent according to certain error criteria. Fortunately the range of parameters is limited by the nature of the problem and by practical considerations of the design. A final selection will take advantage of special points on the day line at equinox and the hour line at noon together with the apriori knowlegde about the parameters.

There are mainly four reasons for the deterioration of the data :
- deficiency of the basic concept,
- deficiencies during the construction of the sundials,
- changes in time by external impact (earthquakes),
- uncertainties of the data acquisition.

The high precision measurements showed that the walls to the South, South-East and East are still very well in place after the long period and add no problems to the analysis. Doubts about the basic concept were listed merely for completeness, thus the analysis for the three sundials mentioned above will have to deal only with the imperfections of the realization.

The first cardinal elements - position of the wall and its orientation with respect to the cardinal points - have been verified. The position of the tower was now found with respect to two trigonometric points as :

$$37^{\circ}\ 58'\ 21''.6\ N\ ;\ 23^{\circ}\ 43'\ 28''.8\ E$$

The orientation of the tower has been determined by observation of the sun and by utilizing information from the global positioning system (GPS) . A coordinate system was defined for the tower based on the best fit of a regular octagon to the inner of the tower at floor level. The cardinal axis of this octogon nearest to the Nord-South-direction has an Azimut of $0^{\circ}\ 05',\ 30''$.

Palaskas, who determined the orientation of several faces outside obtained a mean value of 17′ which shows the imperfection of his measurements not, as he claims, Andronikos', who most likely used the method now known as "Indian circles" as described by Vitruvius (book I,6) obviously to great perfection.

The angle of the ecliptic was known in ancient time with limited accuracy : $\varepsilon = 23^{\circ}51'$ (Eratosthenes). For practical reasons of geometric constructions the value $\varepsilon = 24^{\circ}$ (Vitruvius) might have been used. The value actually changes slowly with time so as to be $\varepsilon = 23^{\circ}42'20''$ in 100 B.C. ($23^{\circ}26'21''$ in 2000 A.C.) .

These differences have only small effects on the shape of the sundials, and one can certainly not hope - as Palaskas suggested - to date the building this way.

The geographic latitude had been determined as the ratio of the shadow length (equinox noon) to gnomon height, expressed with small integer numbers. The following values were reported in case of Athens (with modern writing) :

$$\varphi = \arctan(3/4) = 36^{\circ}52'12''\ \text{Vitruvius (book IX, 7)}$$
$$\varphi = \arctan(16/21) = 37^{\circ}18'14''\ \text{Plinius (book VI, 211)}$$

With the East -and West - sundials the angle of the day line at equinox against the vertical directly represents the latitude. It is found here close to

$$\varphi = \arctan(16/20) = 38^{\circ}39'35''$$

Finally the length of the gnomon is the free parameter to be chosen by the designer for essentially determining the overall size of the sundial. One should reasonably expect "nice" values in terms of the ancient unit of length. As a hypothes is supported, by several dimensions of the tower itself it may be assumed that the construction is based on the roman foot :

$$1\ \text{pes monetalis} = 16\ \text{digiti} = 12\ \text{unciae} = 296.17\ \text{mm}$$

This implies that the tower was built after this length standard had been established in Athens by the Romans. If this result can be confirmed this investigation would also help to answer the question (Freeden) about the time - late 2nd century B.C. or early 1st century B.C. - when the tower was built.

In contrast to modern sundials having a stylus mounted parallel to the axis of the globe [QQ] (Fig.l), only the shadow of the gnomon tip is essential for ancient sundials thus giving freedom for how to mount the gnomon on the wall. In fact the gnomons were mounted here above the horizon and the length is to be measured normal to the

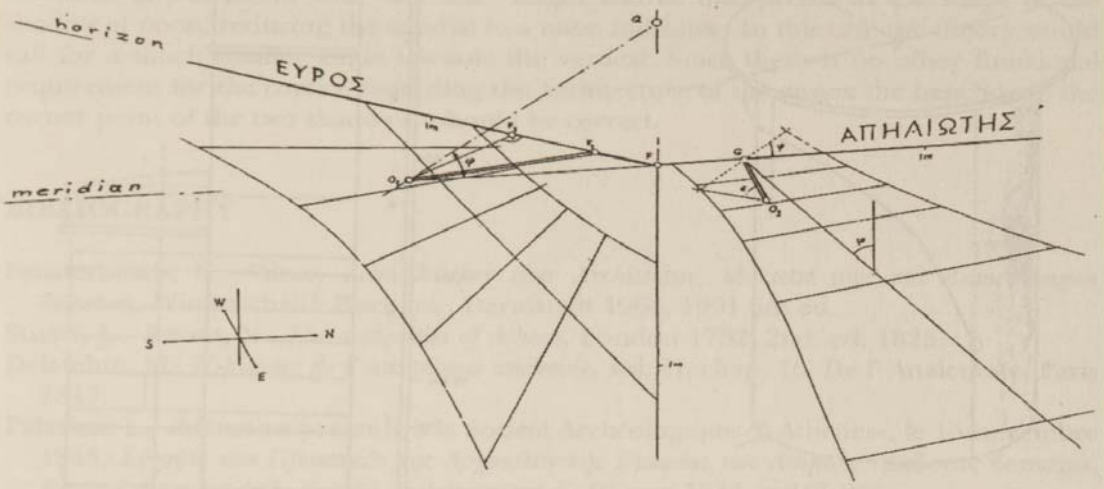


FIG.1 Partial view of the sundials at the South-east and East-face showing different cardinal parameters.

wall [OF] in case of S-, N-, E- and W- dials whereas for the others there is an angle of 30° [OF_2] to the normal (Palaskas is wrong with the NE- / NW- dial giving 45°) .

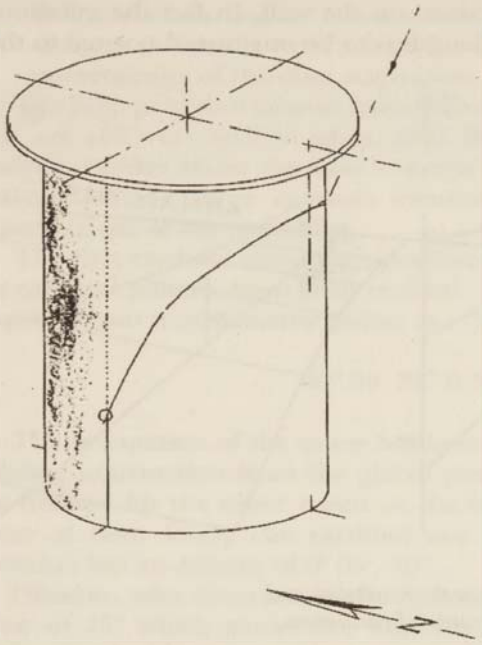
Possibly it can be decided about the question whether the sundials have been constructed according to the theory with certain parameters -as most authors suppose - or just by marking the shadow using another "horologion", maybe the water-clock inside, as a standard (Palaskas). It is remarkable in particular that different values of latitude seem to be implemented. From this it appears plausible that the combination (φ=arctan(4/5), ε=24°) is to be interpreted as geometrical construction, whereas the actual values (37°58'22" / 23°42'20") give a hint to the empirical method. The summary of parameters lists the values of the current working hypothesis.

Finally regarding the cylinder dial Bromley and Wright already pointed out that the remaining lines on the cylinder cannot be explained with gnomons and call for another

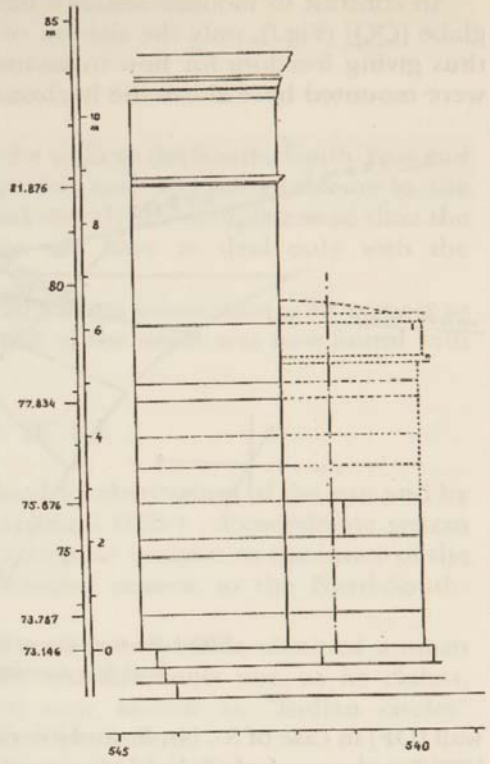
Summary of cardinal elements of the sundials					
	gF		tan φ	φ	ε
	unciae	mm		° ′ ″	° ′ ″
S	11	271.5		37 58 15	23 42 20
SO	30 *	740.4		37 58 15	23 42 20
O	20	493.6	4/5	38 39 35	24 00 00
NO	9 *	222.1		37 58 15	23 42 20
N	11	271.5		37 58 15	23 42 20
NW	9 *	222.1		37 58 15	23 42 20
W	20	493.6	4/5	38 39 35	24 00 00
SW	30 *	740.4		37 58 15	23 42 20

** angle against normal of the wall : 30°*

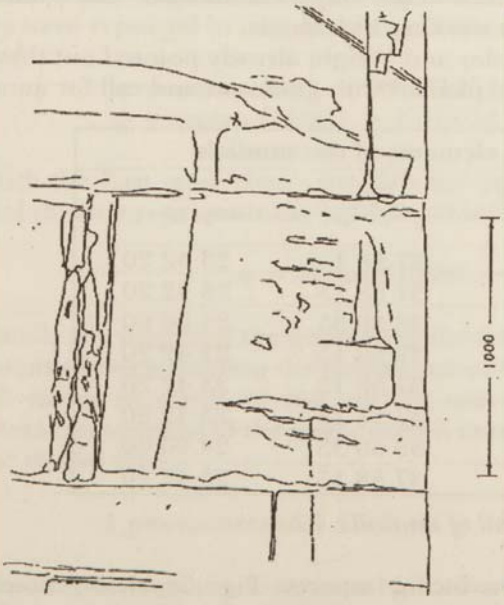
solution. The following proposal has convincing aspects. Fig. 2a shows how the combination of the shadow cast by the cornice like a "brim" of a "hat" together with the "eigen-shadow" of the cylinder body forms a corner point which is unique during the



a



b



c

FIG.2 Sundial on the cylinder of the annex
a) functional principle
b) general view from the west
c) traces of the annex on the south-faceof the tower

day and the seasons. Traces on the South-face (Fig. 2c) lead to the reconstruction of a cornice of 8 unciae depth. With the values (φ=arctan(4/5), ε=24°) this goes well together with the position of the lower end of the noon line on the cylinder at summer solstice. Unfortunately the theory demands that the day line crosses at a right angle with the hour line at noon. The "day line" might also be interpreted as the shape of the shadow at noon, reducing the sundial to a noon indicator. In this case the theory would call for a much smaller angle towards the vertical. Since there is no other functional requirement for the corcice regarding the architecture of the annex the basic idea - the corner point of the two shadows - should be correct.

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ΠΕΡΙΛΗΨΗ

ΑΝΑΛΥΣΗ ΤΩΝ ΗΛΙΑΚΩΝ ΩΡΟΛΟΓΙΩΝ ΣΤΟΝ "ΠΥΡΓΟ ΤΩΝ ΑΝΕΜΩΝ" ΣΤΗΝ ΑΘΗΝΑ (ΠΡΩΤΑ ΑΠΟΤΕΛΕΣΜΑΤΑ ΒΑΣΙΣΜΕΝΑ ΣΕ ΝΕΑ ΔΕΔΟΜΕΝΑ, ΧΑΡΙΣ ΣΕ ΣΥΣΤΗΜΑ ΠΡΟΣΛΗΨΗΣ ΓΕΩΔΑΙΤΙΚΩΝ ΔΕΔΟΜΕΝΩΝ ΜΕΓΑΛΗΣ ΑΚΡΙΒΕΙΑΣ

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Στο παρελθόν, τα ηλιακά ωρολόγια του Πύργου των Ανέμων αποτέλεσαν αντικείμενο ανάλυσης απο τους Delambre (1817), Drecker (1925) και Bromley/Wright (1989), οι οποίοι βασίστηκαν στις μετρήσεις που πήραν το 1762 οι Stuart και Revett. Μόνον ο Παλάσκας (1846) πραγματοποίησε δικές του μετρήσεις. Οι μετρήσεις σε όλες αυτές τις περιπτώσεις δεν είναι πλήρως τεκμηριωμένες και, όπως προκύπτει τώρα απο τις νέες μετρήσεις, δεν είναι τόσο ακριβείς όσο φαίνονταν παλαιότερα. Αυτό έρχεται σε αντίθεση με το γεγονός ότι όλοι οι συγγραφείς καταλήγουν σε ευνοϊκά συμπεράσματα για την ακρίβεια της κατασκευής.

Η πρόσφατη έρευνα ολόκληρου του κτιρίου, η οποία εκτελέστηκε στα πλαίσια ενός Ελληνο-Γερμανικού επιστημονικού προγράμματος, μας παρέχει απόλυτες συντεταγμένες 300 περίπου σημείων, με πιθανή απόκλιση λίγων χιλιοστών για την ανάλυση των ηλιακών ωρολογίων. Θα καταβληθεί προσπάθεια να συμπεριληφθούν και πρακτικές απόψεις περί κατασκευής, όπως η αρχαία γνώση για τις γωνίες του εκλειπτικού και του γεωγραφικού πλάτους, καθώς και η αρχαία μονάδα μήκους. Φαίνεται πολύ πιθανό πως μόνον το Ανατολικό- και το Δυτικό- ηλιακό ωρολόι είχαν κατασκευαστεί βασιζόμενα στη θεωρία, ενώ για τα υπόλοιπα οι σκιές σημειώνονταν χρησιμοποιώντας το προηγούμενο, ίσως απο κοινού με υδραυλικό ρολόι, ως πρότυπο. Θα δοθεί επίσης μια άποψη του μετρολογικού προβλήματος, θέτοντας μια σταθερά απολύτου χρόνου.